

Mathematics

**The Painted Cube problem.
Downloadable resource.**

Mr Millar



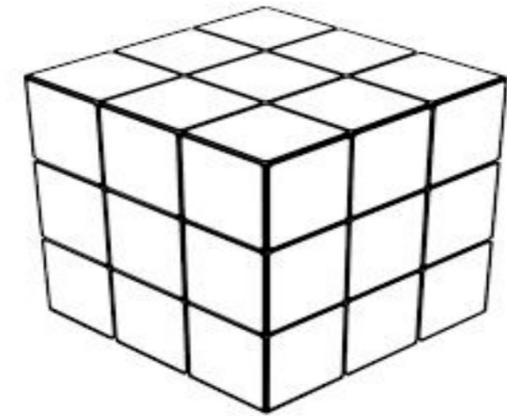
Try this

A 3x3x3 cube is put together using smaller white pieces.

The **outside** is painted red and left to dry.

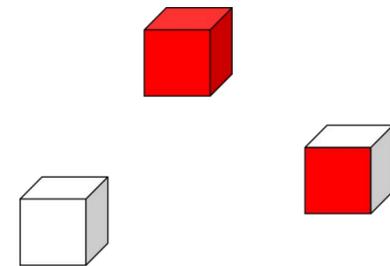
The large cube is then taken apart piece by piece.

What do the **smaller pieces** look like?



I know that the cubes in the corner have _____ faces painted.

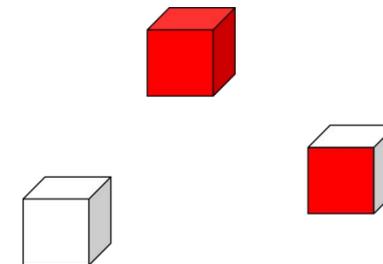
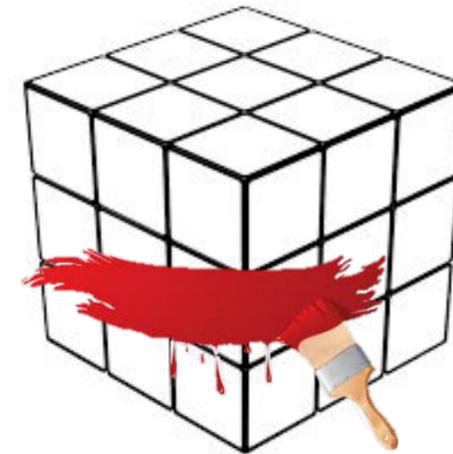
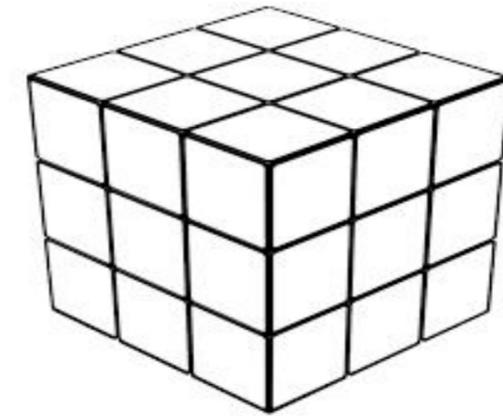
I know how many cubes have 0 faces painted!



Connect

Let's think about the number of cubes with 1 or 2 faces painted.

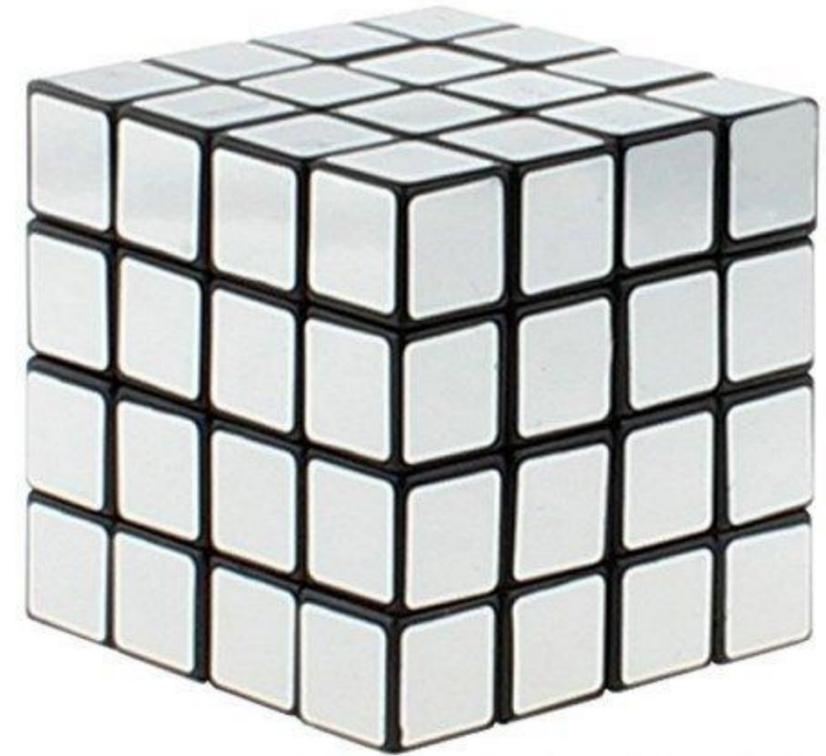
# Faces painted	Where?	# Cubes
3	Corners	8
2		
1		
0	"Inside"	1



Independent task

Now let's think about a 4 x 4 x 4 cube.

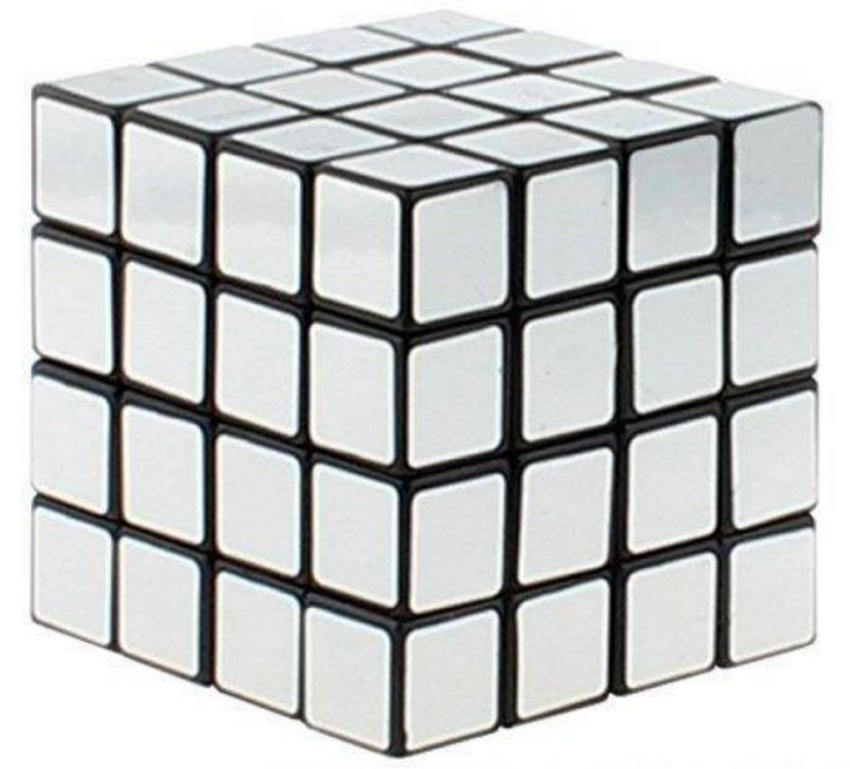
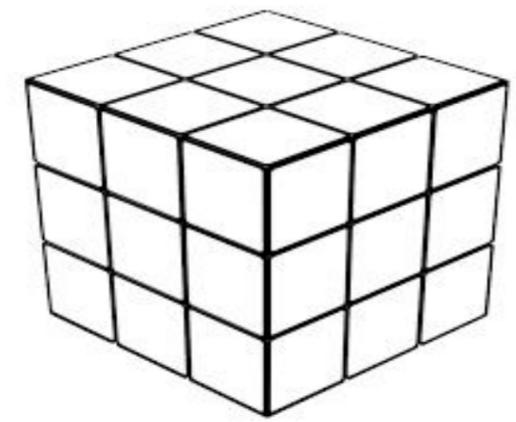
# Faces painted	Where?	# Cubes
3	Corners	
2		
1		
0	"Inside"	



Explore

Now let's think about an $n \times n \times n$ cube.

# Faces painted	Where?	# Cubes
3	Corners	
2		
1		
0	"Inside"	



Answers



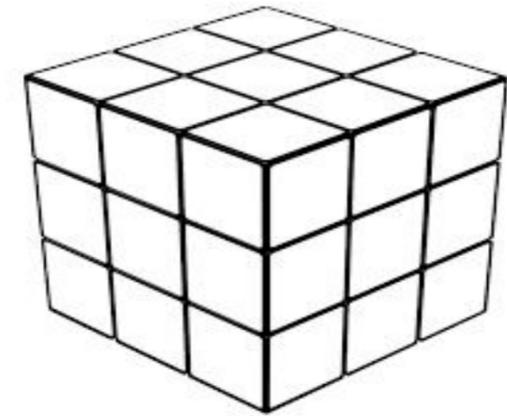
Try this

A $3 \times 3 \times 3$ cube is put together using smaller white pieces.

The **outside** is painted red and left to dry.

The large cube is then taken apart piece by piece.

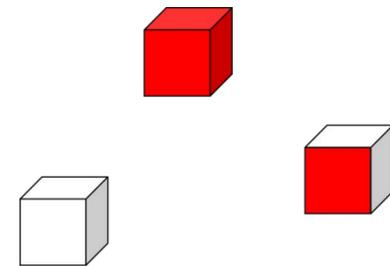
What do the **smaller pieces** look like?



I know that the cubes in the corner have **3** faces painted.

I know how many cubes have 0 faces painted!

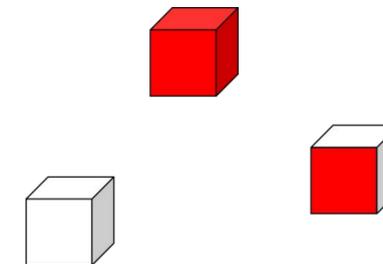
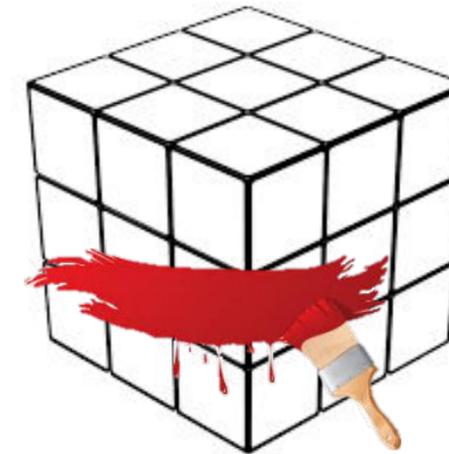
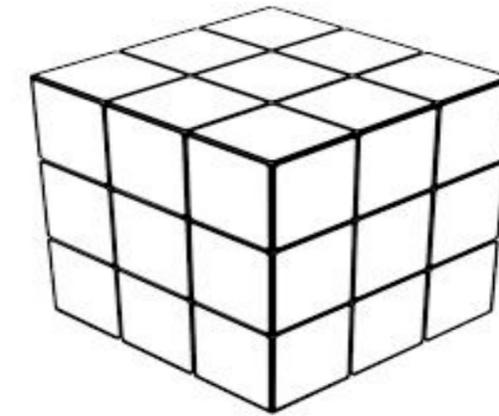
1



Connect

Let's think about the number of cubes with 1 or 2 faces painted.

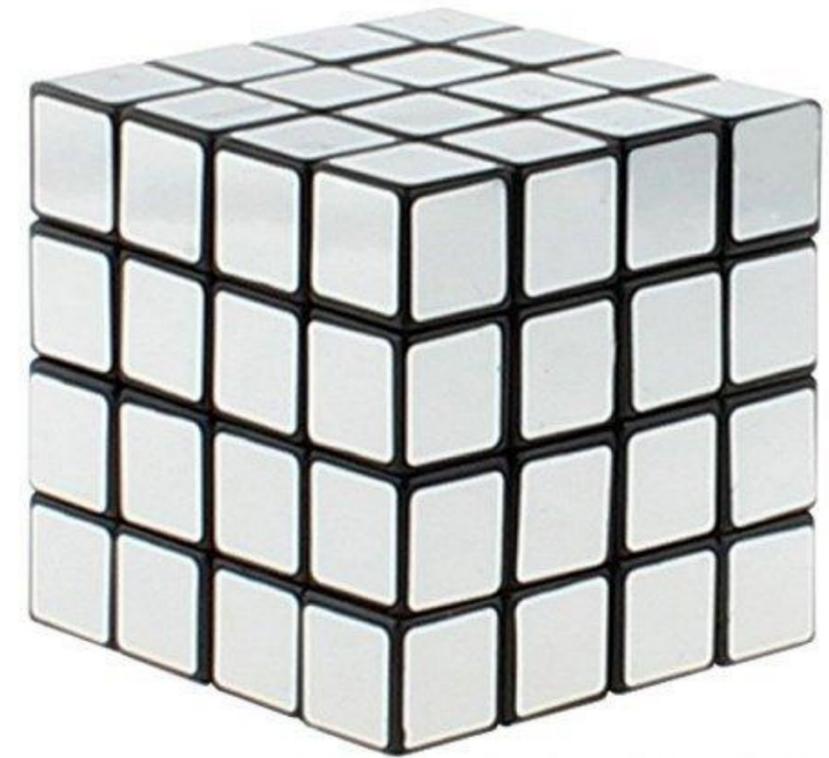
# Faces painted	Where?	# Cubes
3	Corners	8
2	Along the edges (not the corners)	12
1	In the middle of the faces	6
0	"Inside"	1



Independent task

Now let's think about a 4 x 4 x 4 cube.

# Faces painted	Where?	# Cubes
3	Corners	8
2	Along the edges (not the corners)	24
1	In the middle of the faces	24
0	"Inside"	8



Explore

Now let's think about an $n \times n \times n$ cube.

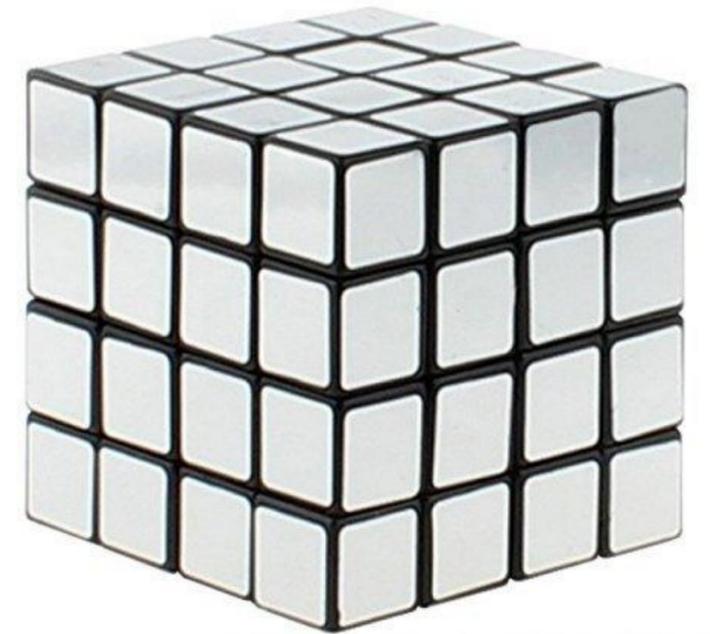
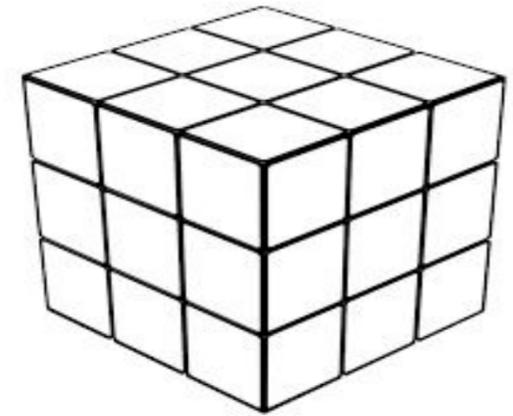
# Faces painted	Where?	# Cubes
3	Corners	8
2	Along the edges (not the corners)	$12n - 24$
1	In the middle of the faces	
0	"Inside"	

On each of the 2 "end" rows, $4n - 8$ faces

On the $(n - 2)$ "middle" rows, 4.

$$2(4n - 8) + 4(n - 2) = 12n - 24$$

6 faces, $(n - 2)^2$ in the middle of each face



Note that these sum to n^3 cubes!

