Maths

Prove an Expression Will Be a Multiple of a Given Number

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Please note some slides do have colour images on them

Prove an expression will be a multiple of a number

1. Prove for all positive integer values of n

a) $(n+5)^2-(n+3)^2$ is always a multiple of 4

b) $(n+9)^2-(n-2)^2$ is always a multiple of 11

2. Prove for all positive integer values of n

a) $(3n+2)^2-(n-2)^2$ is always a multiple of 8

b) $(2n+5)^2-(2n+4)^2+3$ is always a multiple of 4

Prove an expression will be a multiple of a number

3. Prove that the difference between the squares of any two terms in the sequence is always a multiple of 64

20, 28, 36, 44, 52 ...

Prove an expression will be a multiple of a number

4. Take a 3 digit number.

Reverse the digits to form a second 3 digit number.

Prove that the difference between the two 3 digit numbers is a multiple of 9

Answers

Prove an expression will be a multiple of a number

1. Prove for all positive integer values of n

a) $(n+5)^2-(n+3)^2$ is always a multiple of 4

$$(n+5)^2-(n+3)^2 = 4n+16$$
$$= 4(n+4)$$

b) $(n+9)^2-(n-2)^2$ is always a multiple of 11

$$(n+9)^2-(n-2)^2 = 22n+77$$
$$= 11(2n+7)$$

2. Prove for all positive integer values of n

a) $(3n+2)^2-(n-2)^2$ is always a multiple of 8

$$(3n+2)^2-(n-2)^2 = 8n^2+16n$$
$$= 8(n^2+2n)$$

b) $(2n+5)^2-(2n+4)^2+3$ is always a multiple of 4

$$(2n+5)^2-(2n+4)^2+3 = 4n+12$$
$$= 4(n+3)$$

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3. Prove that the difference between the squares of any two terms in the sequence is always a multiple of 64

20, 28, 36, 44, 52 ...

n^{th} term $= 8n + 12$

$$(8n + 12)^2 - (8p + 12)^2 = 64n^2 + 192n - 64p^2 - 192p$$

$$= 64(n^2 + 3n - p^2 - 3p)$$

Prove an expression will be a multiple of a number

4. Take a 3 digit number.

Reverse the digits to form a second 3 digit number.

Prove that the difference between the two 3 digit numbers is a multiple of 9

$$\begin{matrix} xyz \\ zyx \end{matrix} \qquad xyz > zyx$$

$$100x + 10y + z - (100z + 10y + x)$$

$$= 100x + 10y + z - 100z - 10y - x$$

$$= 99x - 99z \ = 9(11x - 11z)$$