

Solve equations numerically : Solve equations using iteration

Maths

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Solve equations numerically : Solve equations using iteration

1. Henry is solving the equation

$$x^2 - 5x - 9 = 0$$

He uses the following iterative equation.

$$x_{n+1} = \sqrt{9 + 5x_n}$$

- Starting with $x_0 = 1$, find the values of x_1 and x_2
- By finding further values of x_n , find a solution to $x^2 - 5x - 9 = 0$ correct to 2 decimal places.

2. Use the iterative formula

$$x_{n+1} = 5 - \frac{2}{x_n^2}$$

to solve the equation $x^3 - 5x^2 + 2 = 0$

Use $x_0 = 3$

Give your answer correct to two decimal places.

What happens if you use $x_0 = -3$ as a starting value?



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3. a) Show that the equation $x^3 - 2x = 1$ can be rearranged as the following equation.

$$x = \sqrt{\frac{1}{x} + 2}$$

b) Write down an iterative formula based on this rearrangement that can be used to solve $x^3 - 2x = 1$

c) Explain why you cannot use $x_0 = -0.2$ as the starting value.

d) Starting with $x_0 = 0.1$ find a solution to $x^3 - 2x = 1$ correct to 2 decimal places.

4. Use the iterative formula below to solve the equation $x^3 - x^2 = 5$

$$x_{n+1} = \sqrt{\frac{5}{x_n} + x_n}$$

Use $x_0 = 4$ and find a solution correct to 3 decimal places.



Answers



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1. Henry is solving the equation

$$x^2 - 5x - 9 = 0$$

He uses the following iterative equation.

$$x_{n+1} = \sqrt{9 + 5x_n}$$

a) Starting with $x_0 = 1$, find the values of x_1 and x_2 $x_1 = 3.741657$ and $x_2 = 5.26386$

b) By finding further values of x_n , find a solution to $x^2 - 5x - 9 = 0$ correct to 2 decimal places. $x = 6.41$

2. Use the iterative formula

$$x_{n+1} = 5 - \frac{2}{x_n^2}$$

to solve the equation $x^3 - 5x^2 + 2 = 0$

Use $x_0 = 3$

Give your answer correct to two decimal places. $x = 4.92$

What happens if you use $x_0 = -3$ as a starting value? $x = 4.92$, you get the same solution



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3. a) Show that the equation $x^3 - 2x = 1$ can be rearranged as the following equation.

$$x = \sqrt{\frac{1}{x} + 2}$$

$$\begin{aligned}x^3 - 2x &= 1 \\x^2 - 2 &= \frac{1}{x} \\x^2 &= \frac{1}{x} + 2 \\x &= \sqrt{\frac{1}{x} + 2}\end{aligned}$$

b) Write down an iterative formula based on this rearrangement that can be used to solve $x^3 - 2x = 1$

$$x_{n+1} = \sqrt{\frac{1}{x_n} + 2}$$

c) Explain why you cannot use $x_0 = -0.2$ as the starting value.

You would end up trying to square root a negative.

d) Starting with $x_0 = 0.1$ find a solution to $x^3 - 2x = 1$ correct to 2 decimal places.

$$x = 1.62$$



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4. Use the iterative formula below to solve the equation $x^3 - x^2 = 5$

$$x_{n+1} = \sqrt{\frac{5}{x_n} + x_n}$$

Use $x_0 = 4$ and find a solution correct to 3 decimal places.

$$x = 2.116$$

