Maths

Odd and Even Number Proofs

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Please note some slides do have colour font on them

Odd and even number proofs

1. Prove that the sum of any two consecutive numbers is odd.

Odd and even number proofs

2. Prove that the difference between the squares of two consecutive odd numbers is even

Odd and even number proofs

3. Prove that $(3x + 3)^2 - (x - 2)^2$ is odd for all integer values of x

Odd and even number proofs

4. Prove that the cube of any odd number is odd.

5. Prove that $(3x)^3 + (4x + 3)^2$ is odd for all integers x

Answers

Odd and even number proofs

1. Prove that the sum of any two consecutive numbers is odd.

$$x + x + 1 = 2x + 1$$

One more than an even number is always odd.

Odd and even number proofs

2. Prove that the difference between the squares of two consecutive odd numbers is even

$$(2x+3)^2-(2x+1)^2 = 4x^2+12x+9-(4x^2+4x+1)$$
$$= 8x+8$$
$$= 2(4x+2)$$

Odd and even number proofs

3. Prove that $(3x + 3)^2 - (x - 2)^2$ is odd

for all integer values of x

$$(3x + 3)^2 - (x - 2)^2 = 9x^2 + 18x + 9 - (x^2 - 4x + 4)$$

$$= 8x^2 + 22x + 5$$

$$= 8x^2 + 22x + 4 + 1$$

$$= \underbrace{2(4x^2 + 11x + 2)}_{\text{Even}} + 1$$

Odd and even number proofs

4. Prove that the cube of any odd number is odd.

$$(2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$$
$$= 2(4x^3 + 6x^2 + 3x) + 1$$

5. Prove that $(3x)^3 + (4x+3)^2$ is odd for all integers x

$$(2x)^3 + (4x+3)^2 = 8x^3 + 16x^2 + 24x + 9$$
$$= 8x^3 + 16x^2 + 24x + 8 + 1$$
$$= 2(4x^3 + 8x^2 + 12x + 4) + 1$$